UNIVERSITY OF DELHI

SCHEME OF EXAMINATION

AND

COURSES OF READING

FOR

A./B.Sc. (HONOURS) EXAMINATION IN MATHEMATICS

Part I Examination 1986
Part II Examination 1987
Part III Examination 1988

Syllabi applicable for students seeking admission to the B.A./B.Sc. (Hons.) Mathematics Course in the academic year 1985-86

Price : Rs 1 - 0

UNIVERSITY OF DELHI

B.A. & B.Sc. (Hons.) Mathematics

SCHEME OF EXAMINATION

The Courses are divided into three parts. Part I is to be covered in the first year, Part II in the second year and Part III in the third year. The contents of the courses have been divided into ten papers. Details of the syllabi for each paper are given in the following pages.

	Duration (Hrs.)	Marks
Part I Examination—1986		
Paper I—Algebra and Analytic Geometry	3	100
Paper II—Calculus	3	100
Part II Examination 1987		
Paper III—Analysis I	3	100
Paper IV—Algebra I	3	100
Paper V—Differential Equations and Mechanics I	3	100
Part III Examination—1988		
Paper VI—Analysis II	3	100
Paper VII—Algebra II	3	100
Paper VIII—Differential Equations and Mechanics II	3	100

Paper IX & X—Any two of the following : 3+3 100+100

(i) Number Theory
(ii) Mathematical Statistics
(iii) Numerical Mathematics
(iv) Linear Programming and Theory of Games
(v) Lattice Theory
(vi) Probability Theory
(vii) Computer Mathematics

Note :—Those who offer Mathematical Statistics as a subsidiary subject *will not be* allowed to offer Mathematical Statistics as an optional course.

Part I Examination (1986)

Paper I—Algebra and Analytic Geometry

Algebra (2/5)

System o complex numbers introduced as a system of ordered pairs of real numbers. Representation of the line segment, straight line, circle and regions in the complex plane. De Moivre's Theorem. Applications to the determination of roots of complex numbers, expression for sin n θ and cos n θ in terms of sin θ and cos θ & vice versa. Sums of simple finite series like Σ cos nθ Σ sin n θ etc.

Algebra of Matrices. Rank of a matrix and its invariance under elementary row and column transformations. Solutions of systems of linear equations with not more than four unkowns.

Symmetric and Skew symmetric matrices. Hermitian and Skew-Hermitian matrices.

Relations between roots and coefficients of a polynomial equation. Evaluation of symmetric functions of roots of cubic and biquadratic equations.

Analytic Geometry (3/5)

Second and higher degree equations representing straight lines. Bisectors of a pair & intersecting lines.

Coaxial circles. Pole and Polar and related properties. Orthogonal circles, Equations of parabola, ellipse and hyperbola in standard forms. Elementry properties of these curves. Change of coordinate axes. Classification of curves represented by an equation of the second degree in two variables.

Analytic study of plane, straight lines, sphere, cone and cylinder. Standard equations of ellipsoid and elementary properties.

Paper II—Calculus 100 Marks

[*Revision Unit* : The real number system, Representation of real numbers as points on the real line, the notion of distance and an interval on the real line. The concept of real functions and their graphs. Derivatives of polynomial and trigonometric functions and of their simple combinations. Primitives of functions and their calculation in simple cases. Integration by substitution and by parts. Applications to determination of areas under plane curves in simple cases.]

Neighbourhood of a point on the real line. Notion of limit of a function, Algebra of Limits. Continuity of a function at a point and on an interval, example of continuous and discontinous functions with geometrical illustrations, algebra of continuous functions, composition of continuous functions.

Derivatives of a function at a point and on an interval, geometrical interpretation of derivative, derivative as a rate measure, algebra of derivable functions, composite functions of derivable functions. Inverse functions and their derivatives, derivative of Implicit functions, and derivatives of function defined parametrically. Derivatives of logarithmic, exponential, and inverse trigonometric function, Hyperbolic and Inverse hyperbolic functions and their derivatives. Derivatives of higher orders and Lebnitz rule.

Tangents and normals, sub-tangents and subnormals, curvature, radius of curvature, circle of curvature, involutes and evolutes and asymptotes of curves in cartesian and polar co-ordinates. Singular points and curve tracing. Functions on R^2 to R, partial differentiation, Euler's theorem on homogenous functions.

Integration by partial fractions, integration of rational and irrational functions. Properties of definite integrals. Reduction formulae.

Evaluation of areas and lengths of curves in the plane, evaluation of volumes and surfaces of solids of revolution.

Part II Examination—1987

Paper III—Analysis I 100 Marks

Sequences of real numbers, convergent sequences, Cauchy sequences, algebras of convergent and Cauchy sequence. Cantor's construction of real numbers (starting from rational numbers). Cauchy's general principle of convergence. Bounded sets of real numbers, suprema, infima, existence of suprema and infima of bounded sets. Monotonic sequences, limit superior, limit inferior of sequences. Infinte series and their convergence. Comparison test, root test, ratio test, Raabe's test, integral test, Leibnitz test and Dirichlet's test for convergence of series. Absolute convergence and rearrangement of of series Convergence and absolute convergence of double series, sufficient conditions for the validity of

$$\sum_{m\,n} a_{m,\,n} = \sum\sum_{m\,n} a_{m,\,n} = \sum\sum_{n\,m} a_{m,\,n}.$$

Products of two absolutely convergent series. Cauchy product of two series, one of which is absolutey convergent. Real numbers and Decimal representations.

Properties of continous functions, uniform continuity, discontinuous functions, types of discontinuity and discontinuty of monotonic functions. Infinite Limits and Limits at infinity.

Rolle's theorem, mean value theorems, Taylor's theorem with Lagrange's and Cauchy's form of remainder, Taylor's and Maclaurin's series of elementary functions. Indeterminate forms.

Functions of two and three variables, their continuity and differentiability, Young's and Schwarz' condition of equality of f_{xy} and f_{yx}. Implicit function theorem. Taylor's theorem and maxima and minima for functions of two variables, Lagrange's method of undetermined multipliers.

Paper IV—Algebra I 100 Marks

Group Theory : Semigroups. groups. different characterizations of groups. Subgroups. Lagrange's Theorem. Cyclic groups. Normal subgroups. Quotient groups. Homomorphism

and Isomorphism theorems. Permutation groups and Cayley's Theorem. Even and odd permutations and A_n.

Rings & Fields : Rings, subrings. ideals. Quotient Rings, Integral domains. Division Rings. Fields. Subfields, Characteristic of a field. Homomorphism and Isomorphism Theorems. Imbedding of a ring without unity in a ring with unity.

Linear Algebra : Vector Spaces. subspaces. Bases and dimension. linear transformations. Algebra of linear transformations. Matrices and linear transformations. Rank and nullity of a linear transformation.

Books Suggested for Reference :

1. Topics in Algebra by I.N. Herstein. Blaisdell Publishing Co—3rd Edition, Vikas Publication 1971.
2. Linear Algebra by K. Hoffman and R. Kunze, Prentice Hall Inc, 1961.
3. Linear Algebra by S. Lang Addison—Wesley Publishing Co. 1968.
4. A First Courses in Abstract Algebra by J.B. Fraleigh, Addison, Wesley Publishing Co.—1968.

Paper V—Differential Equations and Mechanics I 100 Marks

Mechanics :

(Review Unit : Scalar and vector products; projection of a vector on a directed line.)

Triple products. Differentiation and integration of a vector function on an interval. Differentiation of a product of two vectors, Gradient, divergence and curl of a vector. Momeents of a (localised) vector about a point Scalar moment of a vector about a directed line.

Basic concepts of mechanics. Basic laws of mechanics Inertial frames of reference. Work and Energy. Principles of linear momentum, angular momentum and energy for a particle. Conservation field and potential energy. Principle of conservation of energy for a particle.

Rectilinear motion : Uniformly accelerated motion (including connected system). Resisted motion. Harmonic Oscillator. Damped and forced vibrations. Elastic springs and strings. Hooke's law. Vertical and horizontal vibrations of a particle attached to an elastic string.

Motion in a plane : Components of velocity and acceleration : Cartesian, radical and transverse; tangential and normal. Projectile motion in a non-resisting medium. Constrained motion in a horizontal circle, conical pendulum. Constrained motion on a smooth vertical circle. Simple pendulum, Motion of a particle under a central force. Differential equation of a central orbit in both reciprocal polar and pedal coordinates, Newton's law of gravitation and planetary orbits. Kepler's laws of motion deduced from Newton's law of gravitation and vice versa.

Coplaner force systems. Necessary and sufficient condition for equilibrium of a particle. Triangle law of forces, polygon law of forces and Lami's theorem.

Moment of a force about a line. Varignen's theorem for concurrent force systems. Necessary condition for a system of particles to be in equilibrium.

Equipollent force system—definition. Couples, moment of couple, equipollence of two couples. Reduction of a general plane force system. Parallel force systems. Centre of gravity formulae, use of symmetry and standard results (statements only). Principle of virtual work for a system of particles.

Motion of a system of particles in a plane. Motion of the mass centre and motion relative to the mass centre. Principles of linear momentum, angular momentum and energy for a system. Two body problem.

Infinitesimal displacement of a plane lamina. Necessary and sufficient conditions for equilibrium of a rigid body, movable parallel to a fixed plane. Problems on equilibrium under forces including friction (excluding indeterminate cases). Stable equilibrium, Energy test of stability (problems involving one variable only).

Differential Equations :

First order differential equations. Second order differential equations with constant and variable coefficients. Homogeneous linear differential equations. Systems of linear differential equations.

Books Suggested for Reference :

1. Principles of Mechanics by Synge and Griffiths.
2. A text book of Dynamics by F. Chorlton—Chapters 3-6 for problems.
3. Statics by A.S. Ramsey—Chapters 3-6, 9, 11-12 for problems.

Part III Examination—1988

Paper VI—Analysis II

Definition and existence of Riemann integral of a bounded function, Darboux condition of integrability. Riemann integrability of continuous functions and monotonic functions. Riemann integral of functions with finite number of discontinuities and of functions with discontinuity points having a finite number of limit points. Riemann integral as the limit of a sum. The Fundamental theorem of Integral calculus, Mean value theorems. Definition and examples of Riemann-Stieltjes integral of bounded functions.

Sequences and series of functions and their pointwise convergence. Uniform convergence of sequences and series of functions, Weierstrass M-test. Uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, Weierstrass approximation theorem.

Power series and their convergence. Absolute and uniform convergence of a power series. Definitions of exponential, logarithmic, trigonometric functions by means of power series and deduction of their properties.

Fourier series and its convergence. Fourier series of functions of bounded variation and differentiable functions.

Improper integrals. Convergence of an Improper integral, Comparison tests. Dirichlet's test. Beta and Gamma functions, their properties and relationships. Differentiation under Integral sign.

Double and triple integrals, iterated integrals, change of order of Integration, Line, Surface and Volume Integrals. reen's, Gauss' and Stoke's Theorem.

Paper VII—Algebra II 100 Marks

Group Theory : Centre, Normalizer, conjugacy. Class Equation. Finite groups. Cauchy's and Sylow's Theorems. Automorphisms. Inner Automorphisms. Direct product of two groups.

Rings : Imbedding of an interal domain in a field. Field of Quotients. Polynomials over commutative rings. Prime and maximal ideals in commutative rings. Euclidean domains. Principal ideal domains, Unique factorisotion domains Eisenstein's criterion of irreducibility.

Linear Algebra : Dual spaces. transpose of a linear transformation. Direct sum of subspaces. Characteristic values, Characteristic vectors, Cayley-Hamilton Theorem. Inner product space. Gram-Schmidt orthogonalization process.

Books suggested for Reference :

1. Topics in Algebra by I.N. Herstein. Blaisdell Publishing, Co—3rd Edition.
2. Linear Algebra by K. Hoffman and R. Kunze, Prentice Hall Inc. 1961.
3. Linear Algebra by S. Lang, Addison-Wesley Publishing Co. 1968.
4. A First courses in Abstract Algebra by J.B. Fraleigh Addison Wesley Publishing Co—1968.

Paper VIII—Differential Equations and Mechanics II Marks 100

Mechanics :

General force system—total force and total moment relative to a base point Total moment under a change of base

point. Necessary and sufficient conditions for a system to be equipollent to zero. Moment of a couple, composition of couples. Reduction of a force system to a force and a couple. Reduction to a wrench. Invariants of a system.

Euler's theorem (without proof) on displacement of rigid body with one point fixed. General displacement of a rigid body. Infinitesimal displacement of a rigid body about a point. Composition of infinitesimal displacements. Reduction to a screw displacement.

Work done on (i) a particle (ii) a rigid body, in a given infinitesimal displacement. Necessary and sufficient conditions for equilibrium of a rigid body (or a system with workless constraints) by an application for the principle of virtual work.

Motion of a body about a fixed point. Angular velocity. Relation between angular velocity and linear velocity of a point of the body. General motion of a body.

Accelerating frames, Rotating frames. Time of a vector referred to rotating frames. Coriolis force and centrifugal force. Frames with constant angular velocity.

Moment of inertia : definitions and standard results. Momental ellipsoid. Parallel axes and perpendicular axes theorems. Principal axes of inertia, Existence of principal axes of inertia at a point. Determination of principal axes of inertia. Equimomental systems.

Angular momentum and kinetic energy of a rigid body rotating about a fixed point. Kinetic energy of a rigid body in a general motion.

Principles of linear momentum angular momentum and energy for a rigid body. D' Alembert's principle and general equations of motion of rigid body. Motion about a fixed axis. Compound pendulum.

Impulse, impulsiveforces. Impulsive motion in a plane, elastic impact (direct and oblique). Two dimesional problems in rigid body dynamics under finite and impulsive force.

Pressure at a point, Resultant pressure on a plane surface.

Differential Equations :

Total differential eqnations in three variables.

General, singular and complete solutions of partial differential equations of the first order. Lagrange's method, Charpits's method and Monge's method for partial differential equations of the second order. Linear partial differential equations with constant coefficients. Homogeneous linear partial differential equation with variable coefficients.

Books suggested for Reference

1. Principles of Mechanics by Synge and Griffith.
2. A text book of Dynamics by F. Chorlton-Chapters 7-8 for problems.
3. Statics by A.S. Ramsey-Chapter 14 for problems.
4. Hydrostatics by A.S. Ramsey.

Paper IX and X—Any two of the following :

Optional (i) Number Theory

The Basic Representation theorem, Linear Diophantine equations, Fundamental theorem of Arithmetic, Fermat's little theorem and Wilson's theorem.

Basic properties of congruences. Residue system, Euler's Theorem; Chinese Remainder Theorem. Multiplicative arithmetic functions, the function ϕ (n), μ (n) ∂ (n) and their simple properties : Mobius Inversion formula. Primitive roots modulon.

Elementary properties of π (x), Legendre's formula for the highest power of a prime number that divides nl, statement of the prime number theorem, Euler's criterion for quardratic residue, the Legendre symbol, the Quadratic Reciprocity law and its applications.

Sums of two and four squares, Fermat's conjecture. Graphical representation of partitions. Euler's partition theorem.

Books for reference :

G.E. Andrews : 1971, Number Theory.

Option (ii) Mathematical Statistics

Concept of Statistical population and Random sample. Collection and presentation of data, Histogram. Frequency polygon Frequency curves and ogives. Measures of location and dispersion, Moments. Sheppard's corrections for moments upto fourth order. Cumulents. Measures of skewness and kurtosis. Elements of the theory of attributes, Association and Contingency.

Random experiment. Discrete sample space. Events, their union, intersection etc. Probability—Classical. Relative Frequency and axiomatic approaches, Probability spaces. Conditional probability and independence of events. Basic laws of probability. Probability of atleast one event. Geometrical probability. Bayes' theorem. Random variable, Probability function. Probability in continuum. Probability density function and Distribution function. Independent random variables. Mathematical expectations and its laws. Variance and convariance. Bivariate distributions. Marginal and conditional distributions. Correlation and Linear regression for two variables. Rank correlation with Ties. Correlation ratio. Curve fitting by least squares. Multiple and partial correlation for three variates only.

Moment generating function, characteristic function and cumulant function. Bernoulli trials, Distributions : Binomial, Poisson. Normal Geometric, Uniform, Triangular, Exponential, Double exponential, Cauchy Multinomial, Beta and Gamma. Limiting forms of the Binomial and Poisson distributions. Chebychev's lemma, law of Large Numbers. Central limit theorem for identical varities.

Concepts of sampling distribution and standard error. Derivation of sampling distribution of (i) mean of random sample from normal population (ii) sum of squares of standard normal deviates (iii) t and F statistics. Large sample tests for mean and proportion. Tests of significance based on t, F and x^2 (chi-square) statistics.

Books for References :

1. Introduction to Mathematical Statistics by Robert V. Hogg and Allen T. Crag. Macmillan 4th Ed. (1978.
2. Probability & Statistics by Meyer Dwass. Benjamin (1970).
3. Introduction to the theory of Statistics by Alexander M. Mood, Frakin A Graybill and Daune C. Boes. McGraw Hill 3rd Ed. (1970).

Option (iii)—Numerical Mathematics :

Finite, Central and Divided differences. Interpolation. Inverse Interpolation, Numerical differentiation. Numerical Integration; Trapezoidal, Simpson's 1/3rd and 3/8th rules, Weddle's Rule Gauss quadrature formula of integration, Geogory's formula and the Euler Maclaurin's formula.

Solution of difference equation of the first order. General equations. Linear difference equations with contact coefficients.

Solution of ordinary differential equations-One step methods : Euler's modified, Picard's, Runge Kutta's methods. Method of starting the solution and continuing the solution : Adams, Adams Bashforth, Milne.

Simultaneous linear epuations : Gauss elimination, Gauss Seidel Jordad's and Relaxation methods (Simple problems).

Finding roots of polynormial equations : Regula Falsi, Bisection, Newton Raphson method for several variables, iterative method and its generalisation. Chebychev's Bigre-Vieta, Lin-Baristow's Graeffe's Root Squaring methods and their convergence.

Significant figures and errors of computation. Nomograms

Books for References:

1.	Froberg	Numerical Analysis
2.	Kunz	Numerical Analysis
3.	Nielson	Numerical Analysis

4.	Levy and Lessman	Finite Difference equations (Chapters 3 and 4).
5.	Hildebrand	Introduction to Numerical Analysis

Option (iv)—Linear Programming and Theory of Games :

Linear Programming : Convex sets and their properities. Theory of simplex method. Revised simplex algorithm. Degeneracy Duality theory. Sensitivity Analysis, Parametric linear programming. Transportation and Assignment problems.

Theory of Games : Rectangular Games. Saddle points. Mixed strategies. Fundamental Theorem for rectangular games. Properties of optimal strategies. Relations of dominance. Various methods for solving rectangular games, Inter-relation between the theory of games and linear programming.

Books for References :

1.	G. Hadley	: Linear Programming
2.	S. I. Gass	: Linear Programmin : Methods Applications.
3.	McKinsey	: Introduction to the TEeory of Games.

Option (v)—Lattice Theory :

Partial order, Chains, Lattices, Examples of Lattices. Meets and Joins, Duality. Length and Covering conditions. Atoms, complements. Complemented and relatively complemented Lattices. Sublattices, Modular and semi-modular lattices. Lattices of groups and modules.

Distributive lattices. Irreducible elements. Ideals of a lattice, Homomorpbism. Isomorphism. Dual Isomorphisim. Boolean Algebras.

Books for Reference :

Thomas Donnellan : Lattice Theory, Pergamon Press, Oxford, 1968.

(Chapter 2, 3, 4 ; Chapter 5, sections 21, 22 and 23 only Chapter 6 ; sections 25, 26 and 27 only).

Option (vi)—Probability Theory :

Probability spaces. Finite Probability space. Conditional Probability, Bayes' theorem, Random variables. Mathematical Expectation and Moments. Joint Distributions, Independent Random variables. Convergence of sequence of random variablecs, convergence in distribution, convergence in probability, almost sure convergence, convergence in quadratic mean, Helley-Bray Theorem. Complex valued Random variables. Characteristic function, Invrsion theorem. Continuity theorem. Distribution of X and S. Kolmogorov's inequality. Weak and strong laws of Large Numbers.

Books for References :

1. Modern Probability. Theory and its applications : E. Parzon.
2. An Introduction to Probability Theory and its applications Vol. I (3rd edition). W. Feller.
3. Probability. Elements of the Mathematics Theory: E. R. Heathcote.

Option (vii)—Computer Mathematics :

Finite State Mechanics ; Binary devices and states, finite state mechanics, converting and equivalence, equivalent states, minimization procedure. Turning machine.

Programming Languages : Arithmetic expression, identifiers, arrays, FORMAT statements, Block structures in ALGOL, The ALGOL grammer.

Boolean Algebras : Boolean polynomials, Block g diagram for getting network, connection with logical capabilities of ALGOL. Boolean applications, subalgebras, disjunctive normal form.

Optimization and Computer Design ; Optimization, Computerizing optimization, Logic design, NAND gates and

NOR gates, the mininization problems, procedure for deriving prime implicants, Flip flops, sequential machine, design.

Binary Group Codes : Encoding and decoding, Block codes, matrix encoding techniques, Group codes : Decoding tables, Hamming codes.

Bose-Chaudhury-Hoequengnen Codes : Computations in GF (2"), BCH Codes.

Modern Applied Algebra by Garrot Birkhoff and Thrnas C. Bartee, 1970, (McGraw Hill).

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